

Chapitre 5: Structures de données

INF3135

Construction et maintenance de logiciels

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Généralités

Structure de données

Organisation **logique** d'un ensemble de données

Plusieurs objectifs

- **Simplifier** le traitement
- Offrir des opérations **efficaces**
- Économiser de l'espace **mémoire**

Interface et implémentation

- **Interface**: opérations supportées (type abstrait)
- **Implémentation**: organisation des données en mémoire, actions effectuées pour réaliser les opérations

Type abstrait: exemples

- **Pile** (*stack*): principe *last in first out* (*LIFO*)
- **File** (*queue*): principe *first in first out* (*FIFO*)
- **File à priorité** (*priority queue*): la priorité des éléments peut être augmentée ou diminuée
- **Liste**: les éléments sont ordonnés, on peut avoir des opérations d'accès, d'insertion, de suppression
- **Ensemble** (*set*): aucune donnée répétée (doublet), vérification d'appartenance d'éléments, données ordonnées ou non
- **Tableau associatif** (*map*): un ensemble de paires clé-valeur, les clés doivent être uniques, les valeurs peuvent être répétées
- **Partition**: division d'un ensemble en parties, fusion entre parties, vérifier si deux éléments sont dans la même partie
- **Graphe**: relations symétriques (graphes non orientés) ou non symétrique (graphes orientés) entre entités

Implémentation: exemples

- **Tableau statique**: mémoire allouée et fixe, capacité maximale permise
- **Tableau dynamique**: tableau compressé ou allongé selon les besoins
- **Liste simplement chaînée**: chaque élément a une référence au suivant
- **Liste doublement chaînée**: chaque élément a une référence à l'élément précédent et à l'élément suivant
- **Structure arborescente**: arbres binaires, arbres préfixes, arbres suffixes, arbres d'arité quelconque, arbres coloriés, arbres-kd
- **Tableau multidimensionnel**: statiques ou dynamiques
- **Liste d'adjacence**: matrice creuse, graphes

Invariants et opérations

Invariant

- **Propriété** qui doit être satisfaite en tout temps
- Généralement vérifiable à l'aide d'une **fonction** booléenne

Opération

- Toute fonction qui **modifie** la structure de données
- Doit toujours préserver les **invariants**

Exemples

- **Chaîne de caractères**: termine par `'\0'`
- **Liste simplement chaînée**: le dernier noeud pointe vers `NULL`
- **Arbre binaire de recherche**: les clés respectent l'ordre

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Allocation dynamique

Allocation dynamique

- Jusqu'à maintenant: mémoire réservée de façon **statique**
- Or, cette information n'est **pas toujours connue** à l'avance
- **Solution**: allouer l'espace mémoire de façon **dynamique**
- Dans la bibliothèque `stdlib.h`:

```
// Réserve un bloc de taille `size`  
void* malloc(size_t size);  
// Libère l'espace mémoire pointé par `ptr`  
void free(void* ptr);  
// Réserve un bloc de taille `nmemb * size` initialisé à 0  
void* calloc(size_t nmemb, size_t size);  
// Redimensionne un bloc de taille `size` déjà alloué dynamiquement  
void* realloc(void* ptr, size_t size);  
// Redimensionne un bloc de taille `nmemb * size`  
void* reallocarray(void* ptr, size_t nmemb, size_t size);
```

Les fonctions `malloc` et `calloc`

```
void* malloc(size_t size);
```

- Réserve sur le **tas** (*heap*) un bloc de mémoire
- De **taille** `size`
- **Retourne** un pointeur vers ce bloc
- **Retourne** `NULL` s'il n'y a plus d'espace mémoire

```
void* calloc(size_t nmemb, size_t size);
```

- Réserve `nmemb` blocs de mémoire **consécutifs**
- De taille **individuelle** `size`
- **Initialise** toutes les valeurs à 0
- **Retourne** un pointeur vers ce bloc
- **Retourne** `NULL` s'il n'y a plus d'espace mémoire

La fonction `free`

```
void free(void* ptr);
```

- **Libère** l'espace mémoire pointé par `ptr`
- Réservé lors d'un appel **précédent** à `malloc` ou `calloc`
- La taille libérée est **égale** à celle réservée
- Si `ptr == NULL`, alors rien ne se passe

Attention

- Si `free` a déjà été appelé sur `ptr`
- Ou si la mémoire pointée par `ptr` n'a pas été allouée précédemment

Alors le comportement est **indéfini**.

Les fonctions `realloc` et `reallocarray`

```
void* realloc(void* ptr, size_t size);
```

- **Redimensionne** un bloc de mémoire à la taille `size`
- **Préalablement** réservé avec `malloc` ou `calloc`
- **Retourne** un pointeur vers le bloc redimensionné
- **Retourne** `NULL` s'il n'y a plus d'espace mémoire

```
void* reallocarray(void* ptr, size_t nmemb, size_t size);
```

- **Équivalent** à `realloc(ptr, nmemb * size)`

Attention

- La valeur des **octets** présents avant et après est **préservée**
- Si **agrandissement**, les « nouveaux » octets sont **indéterminés**
- Pointeur retourné peut être **différent** du pointeur en 1er argument

Gestion de la mémoire

Fuite de mémoire (*memory leak*)

- Mémoire **réservée** mais non **référéncée**
- Provoquée lorsqu'on appelle `malloc` ou `calloc`
- Et qu'on oublie de libérer avec `free`
- **Attention**: souvent « cachée » derrière une autre fonction

Exemples

- Initialisation d'une **structure de données** dynamique
- Utilisation de la fonction `strdup` (duplication de chaîne)
- Bibliothèque **SDL**: `SDL_Init`

Comment les éviter?

- S'assurer que tout **appel** à `malloc` ou `calloc`
- Est **couplé** à un appel de la fonction `free`
- Souvent à l'aide de **fonctions**

Responsabilité de mémoire

- Si une fonction utilise `malloc` sans `free` associé
- Le comportement doit être **documenté** (*docstring*)
- **Exemple**: la fonction `strdup`
The `strdup()` function returns a pointer to a new string which is a duplicate of the string `s`. Memory for the new string is obtained with `malloc(3)`, and can be freed with `free(3)`.
- Fournir une fonction **complémentaire** qui libère l'espace alloué
- **Exemple**: `SDL_Quit` est l'« inverse » de `SDL_Init`

Attention

- Pas de **ramasse-miettes** (*garbage collector*)
- Préférer un passage par **adresse**
- Et utiliser `malloc/calloc/free` seulement lorsqu'inévitable

L'outil Valgrind

- Cadriciel permettant de concevoir des outils d'**analyse dynamique**
- Permet de détecter des erreurs de **gestion de mémoire**
- Et de **profiler** un programme en détail
- **Lien officiel:** <http://valgrind.org/>
- **Invocation:**

```
$ valgrind [options valgrind] [programme] [options programme]
```

Plusieurs dizaines d'**options**:

- `--tool=<toolname>`: outil (par défaut, memcheck)
- `--leak-check=<no|summary|yes|full>`: vérifier fuites mémoires
- `--time-stamp=<yes|no>`: afficher chronologie
- `--track-origins=<yes|no>`: origine des valeurs non initialisées
- etc.

Exemple (1/5)

```
#include <stdio.h>
#include <stdlib.h>

double* sum(const double* v1, const double* v2, unsigned int n) {
    double* v = malloc(n * sizeof(double));
    for (unsigned int i = 0; i < n; ++i)
        v[i] = v1[i] + v2[i];
    return v;
}

void print_vector(const double* v, unsigned int n) {
    printf("[ ");
    for (unsigned int i = 0; i < n; ++i)
        printf("%1f ", v[i]);
    printf("]");
}

int main(void) {
    double v1[] = { 2.0, -1.5, 3.4};
    double v2[] = {-1.0, 2.1, -0.8};
    print_vector(sum(v1, v2, 3), 3);
    return 0;
}
```

Résultat:

```
[ 1.000000 0.600000 2.600000 ]
```

Exemple (2/5)

```
$ gcc sum_malloc.c -o sum_malloc
$ valgrind ./sum_malloc
$ valgrind --leak-check=yes ./sum_malloc
==6724== Memcheck, a memory error detector
==6724== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==6724== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==6724== Command: ./sum_malloc
==6724==
[ 1.000000 0.600000 2.600000 ]==6724==
==6724== HEAP SUMMARY:
==6724==     in use at exit: 24 bytes in 1 blocks
==6724==   total heap usage: 2 allocs, 1 frees, 1,048 bytes allocated
==6724==
==6724== 24 bytes in 1 blocks are definitely lost in loss record 1 of 1
==6724==    at 0x4C2FB0F: malloc (in /usr/lib/valgrind/[...])
==6724==    by 0x10876B: sum (in [...]/code/sum_malloc)
==6724==    by 0x1088BE: main (in [...]/code/sum_malloc)
==6724==
==6724== LEAK SUMMARY:
==6724==     definitely lost: 24 bytes in 1 blocks
==6724==     indirectly lost: 0 bytes in 0 blocks
==6724==     possibly lost: 0 bytes in 0 blocks
==6724==     still reachable: 0 bytes in 0 blocks
==6724==     suppressed: 0 bytes in 0 blocks
==6724==
==6724== For counts of detected and suppressed errors, rerun with: -v
==6724== ERROR SUMMARY: 1 errors from 1 contexts (suppressed: 0 from 0)
```

Exemple (3/5)

```
#include <stdio.h>
#include <stdlib.h>

double* sum(const double* v1, const double* v2, unsigned int n) {
    double* v = malloc(n * sizeof(double));
    for (unsigned int i = 0; i < n; ++i)
        v[i] = v1[i] + v2[i];
    return v;
}

void print_vector(const double* v, unsigned int n) {
    printf("[ ");
    for (unsigned int i = 0; i < n; ++i)
        printf("%lf ", v[i]);
    printf("]");
}

int main(void) {
    double v1[] = { 2.0, -1.5, 3.4};
    double v2[] = {-1.0, 2.1, -0.8};
    double* v = sum(v1, v2, 3);
    print_vector(v, 3);
    free(v);
    return 0;
}
```

Exemple (4/5)

```
$ gcc sum_malloc_free.c -o sum_malloc_free
$ valgrind --leak-check=yes ./sum_malloc_free
==10376== Memcheck, a memory error detector
==10376== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==10376== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==10376== Command: ./sum_malloc_free
==10376==
[ 1.000000 0.600000 2.600000 ]
==10376==
==10376== HEAP SUMMARY:
==10376==       in use at exit: 0 bytes in 0 blocks
==10376==   total heap usage: 2 allocs, 2 frees, 1,048 bytes allocated
==10376==
==10376== All heap blocks were freed -- no leaks are possible
==10376==
==10376== For counts of detected and suppressed errors, rerun with: -v
==10376== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
```

Allocation dynamique

- Est-ce que malloc est vraiment nécessaire ici?
- **Réponse:** non!

Exemple (5/5)

```
#include <stdio.h>
#include <stdlib.h>

void compute_sum(const double* v1, const double* v2,
                 double* v, unsigned int n) {
    for (unsigned int i = 0; i < n; ++i)
        v[i] = v1[i] + v2[i];
}

void print_vector(const double* v, unsigned int n) {
    printf("[ ");
    for (unsigned int i = 0; i < n; ++i)
        printf("%1f ", v[i]);
    printf("]");
}

int main(void) {
    double v1[] = { 2.0, -1.5, 3.4};
    double v2[] = {-1.0, 2.1, -0.8};
    double v[3];
    compute_sum(v1, v2, v, 3);
    print_vector(v, 3);
    return 0;
}
```

Piles

Pile

- Structure de donnée **fondamentale**
- Stratégie *LIFO* = *last in first out*

Interface minimale (pile de caractères)

```
// Initialize the stack
void stack_initialize(Stack* s);
// Is stack empty?
bool stack_is_empty(const Stack* s);
// Push a value on top
void stack_push(Stack* s, char value);
// Pop the value from the top
char stack_pop(Stack* s);
// Delete a stack
void stack_delete(Stack* s);
```

Implémentation

- **Tableau** statique ou dynamique
- Liste **simplement chaînée**

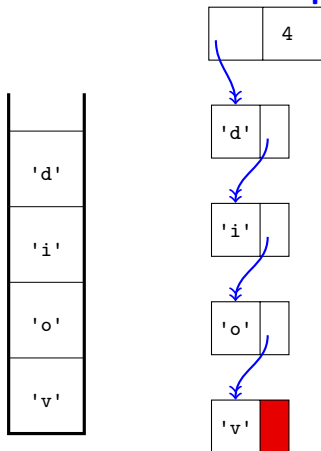
Représentation

Déclaration de types en C:

```
// Node
struct StackNode {
    char value;
    struct StackNode* next;
};

// Stack
typedef struct {
    struct StackNode* first;
    unsigned int size;
} Stack;
```

Représentation schématique:



Invariants

Trois invariants

Pour toute **pile** `s` et pour tout **noeud** `node` de `s`,

- `node.next == NULL` **ssi** `node` est le dernier noeud de `s`
- `s.first == NULL` **ssi** `stack_is_empty(s)` est *vrai*
- Le nombre de noeuds dans `s` est donné par `s.size`

Utilité?

- Quand on implémente `stack_push` et `stack_pop`
- **Supposer** les invariants satisfaits en **début de fonction**
- Et s'assurer qu'ils sont encore satisfaits à la **fin**

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}

// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}

// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

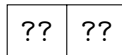
```
// Initialize the stack
```

```
void stack_initialize(Stack* s) {
```

```
    s->first = NULL;
```

```
    s->size = 0;
```

```
}
```



```
// Is stack empty?
```

```
bool stack_is_empty(const Stack* s) {
```

```
    return s->size == 0;
```

```
}
```

```
// Push a value on top
```

```
void stack_push(Stack* s, char value) {
```

```
    struct StackNode* node = malloc(sizeof(struct StackNode));
```

```
    node->value = value;
```

```
    node->next = s->first;
```

```
    s->first = node;
```

```
    ++s->size;
```

```
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}
```



```
// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}
```

```
// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}
```



```
// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}
```

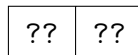
```
// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}
```



```
// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}
```



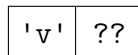
```
// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}
```



```
// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}
```



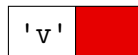
```
// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}
```



```
// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}
```



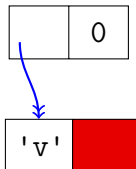
```
// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}

// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}

// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```

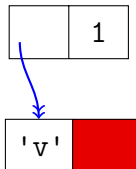


Implémentation (1/2)

```
// Initialize the stack
void stack_initialize(Stack* s) {
    s->first = NULL;
    s->size = 0;
}

// Is stack empty?
bool stack_is_empty(const Stack* s) {
    return s->size == 0;
}

// Push a value on top
void stack_push(Stack* s, char value) {
    struct StackNode* node = malloc(sizeof(struct StackNode));
    node->value = value;
    node->next = s->first;
    s->first = node;
    ++s->size;
}
```



Implémentation (2/2)

```
// Pop the value from the top
char stack_pop(Stack* s) {
    if (!stack_is_empty(s)) {
        char value = s->first->value;
        struct StackNode* node = s->first;
        s->first = node->next;
        free(node);
        --s->size;
        return value;
    } else {
        fprintf(stderr, "Cannot pop from empty stack\n");
        exit(1);
        return '?';
    }
}

// Delete a stack
void stack_delete(Stack* s) {
    while (!stack_is_empty(s)) stack_pop(s);
}
```

Exemple d'utilisation: parenthésage équilibré

```
/**
 * Returns true if and only if an expression is balanced
 *
 * @param expr The expression to check
 * @returns True if and only if the expression is balanced
 */
bool is_balanced(const char* expr) {
    bool balanced = true;
    Stack s;
    stack_initialize(&s);
    for (unsigned int i = 0; balanced && expr[i] != '\0'; ++i) {
        if (expr[i] == '(') {
            stack_push(&s, '(');
        } else if (expr[i] == '[') {
            stack_push(&s, '[');
        } else if (expr[i] == '{') {
            stack_push(&s, '{');
        } else if (expr[i] == ')' || expr[i] == ']' || expr[i] == '}') {
            if (stack_is_empty(&s))
                balanced = false;
            else
                balanced = expr[i] == stack_pop(&s);
        }
    }
    balanced = balanced && stack_is_empty(&s);
    stack_delete(&s);
    return balanced;
}
```

Tableaux dynamiques

Tableau dynamique

- Tableau dont la taille **varie** selon l'usage
- Supporté **par défaut** dans plusieurs langages
- **Exemples**: C++ (vector), Java (ArrayList), Python (listes)

Implémentation

- **Capacité** (*capacity*): taille « réelle » du tableau en mémoire
- **Taille** (*size*): nombre d'éléments « pertinents »

Redimensionnement

- **Automatiquement**, en doublant la taille
- Ou **manuellement**, par un appel de fonction
- Parfois, **contracté** automatiquement quand trop d'espace inoccupé

Interface

```
// Initialize an empty array
void array_initialize(Array* a);

// Append an element to the end of an array
void array_append(Array* a, int e);

// Insert an element in an array
void array_insert(Array* a, unsigned int i, int element);

// Remove an element from an array at a given index
void array_remove(Array* a, unsigned int i);

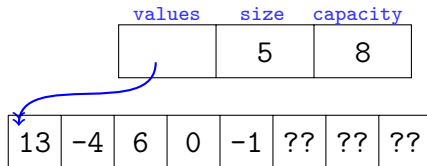
// Check if an array contains a given element
bool array_has_element(const Array* a, int e);

// Return the element at a given index in an array
int array_get(const Array* a, unsigned int i);

// Delete an array
void array_delete(Array* a);
```

Représentation

```
// A resizable array
typedef struct {
    int* values;           // The values of the array
    unsigned int size;     // The current size
    unsigned int capacity; // The capacity
} Array;
```



Invariants:

- La taille réservée par `values` est `capacity * sizeof(int)`
- `size <= capacity`
- Seules les `size` premières valeurs sont « pertinentes »

Implémentation (1/4)

```
// Initialize an empty array
void array_initialize(Array* a) {
    a->values = malloc(sizeof(int));
    a->size = 0;
    a->capacity = 1;
}

// Append an element to the end of an array
void array_append(Array* a, int e) {
    array_resize_if_needed(a);
    a->values[a->size] = e;
    ++a->size;
}

// Insert an element in an array
void array_insert(Array* a, unsigned int i, int e) {
    array_check_index(a, i);
    array_resize_if_needed(a);
    for (unsigned int j = a->size - 1; j > i; --j)
        a->values[j] = a->values[j - 1];
    a->values[i] = e;
    ++a->size;
}
```

Implémentation (2/4)

```
// Remove an element from an array at a given index
void array_remove(Array* a, unsigned int i) {
    array_check_index(a, i);
    ++i;
    while (i < a->size) {
        a->values[i - 1] = a->values[i];
        ++i;
    }
    --a->size;
}

// Check if an array contains a given element
bool array_has_element(const Array* a, int e) {
    unsigned int i = 0;
    while (i < a->size && array_unsafe_get(a, i) != e)
        ++i;
    return i < a->size;
}
```

Implémentation (3/4)

```
// Return the element at a given index in an array
int array_get(const Array* a, unsigned int i) {
    array_check_index(a, i);
    return array_unsafe_get(a, i);
}

// Delete an array
void array_delete(Array* a) {
    free(a->values);
}

// Resize an array if the capacity is reached
void array_resize_if_needed(Array* a) {
    if (a->size == a->capacity) {
        a->capacity *= 2;
        a->values = realloc(a->values, a->capacity * sizeof(int));
    }
}
```

Implémentation (4/4)

```
// Return the element at index i in an array (no check)
int array_unsafe_get(const Array* a, unsigned int i) {
    return a->values[i];
}

// Print an array to stdout
void array_print(const Array* a) {
    unsigned int i;
    printf("[");
    for (i = 0; i < a->size; ++i) {
        printf(" %d", a->values[i]);
    }
    printf(" ]");
}

// Check if an index is out of bound
void array_check_index(const Array* a, unsigned int i) {
    if (i >= a->size) {
        fprintf(stderr, "Invalid index %d (size = %d)\n", i, a->size);
        exit(1);
    }
}
```

Utilisation (1/2)

```
int main() {
    Array a; array_initialize(&a);
    printf("Appending 3, 2, 5, 7, 8, 7: ");
    array_append(&a, 3); array_append(&a, 2);
    array_append(&a, 5); array_append(&a, 7);
    array_append(&a, 8); array_append(&a, 7);
    array_print(&a);
    printf("\nRemoving at position 2: ");
    array_remove(&a, 2); array_print(&a);
    printf("\nRemoving at position 4: ");
    array_remove(&a, 4); array_print(&a);
    printf("\nRemoving at position 2: ");
    array_remove(&a, 2); array_print(&a);
    printf("\nInserting 7 at position 1: ");
    array_insert(&a, 1, 7); array_print(&a);
    printf("\n");
    for (int e = 0; e <= 9; e += 2)
        printf("Has element %d ? %s\n", e,
            array_has_element(&a, e) ? "yes" : "no");
    array_delete(&a);
}
```

Utilisation (2/2)

Résultat:

Appending 3, 2, 5, 7, 8, 7: [3 2 5 7 8 7]

Removing at position 2: [3 2 7 8 7]

Removing at position 4: [3 2 7 8]

Removing at position 2: [3 2 8]

Inserting 7 at position 1: [3 7 2 8]

Has element 0 ? no

Has element 2 ? yes

Has element 4 ? no

Has element 6 ? no

Has element 8 ? yes

Tableaux multidimensionnels

Tableau multidimensionnel

- Généralisation d'un tableau à **plusieurs dimensions**
- Aussi appelé **matrice**
- Simplifie la manipulation des **données** homogènes
- En les organisant selon leurs **dimensions**

Version aplatie

- Tableau **unidimensionnel**
- **Plus** compact
- Mais on doit gérer l'**accès** (indexation)

Par indirection

- Tableau de **pointeurs**
- **Moins** compact
- Mais plus facile de gérer l'indexation

Interface (partielle)

```
// Initialize a matrix
void matrix_initialize(struct Matrix* m,
                      unsigned int r,
                      unsigned int c,
                      bool random);

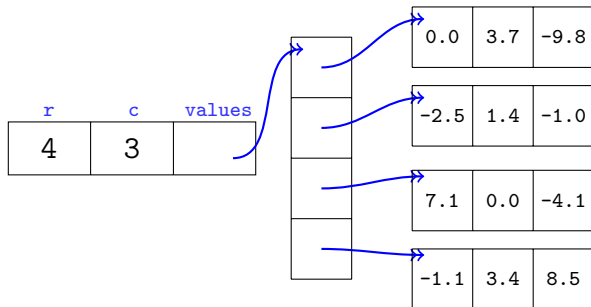
// Print a matrix to stdout
void matrix_print(const struct Matrix* m);

// Add the matrix `m2` to the matrix `m1`
void matrix_add(struct Matrix* m1, const struct Matrix* m2);

// Delete the given matrix
void matrix_delete(struct Matrix* m);
```

Représentation

```
struct Matrix {  
    unsigned int r; // Number of rows  
    unsigned int c; // Number of columns  
    double** values; // Values in matrix  
};
```



Implémentation (1/3)

```
// Initialize a matrix with 0 or random values
void matrix_initialize(struct Matrix* m,
                      unsigned int r,
                      unsigned int c,
                      bool random) {
    m->r = r;
    m->c = c;
    m->values = malloc(r * sizeof(double*));
    for (unsigned int i = 0; i < r; ++i) {
        m->values[i] = malloc(c * sizeof(double));
        for (unsigned int j = 0; j < c; ++j) {
            if (random)
                m->values[i][j] = (float)rand() /
                                   (float)(RAND_MAX / 20.0) - 10.0;
            else
                m->values[i][j] = 0.0;
        }
    }
}
```

Implémentation (2/3)

```
// Print a matrix to stdout
void matrix_print(const struct Matrix* m) {
    for (unsigned int i = 0; i < m->r; ++i) {
        printf("[ ");
        for (unsigned int j = 0; j < m->c; ++j)
            printf("%6.2lf ", m->values[i][j]);
        printf("]\n");
    }
}

// Add the second matrix to the first one
void matrix_add(struct Matrix* m1, const struct Matrix* m2) {
    if (m1->r != m2->r || m1->c != m2->c) {
        fprintf(stderr, "Error: matrices have different dimensions\n");
        exit(1);
    }
    for (unsigned int i = 0; i < m1->r; ++i)
        for (unsigned int j = 0; j < m1->c; ++j)
            m1->values[i][j] += m2->values[i][j];
}
```

Implémentation (3/3)

```
// Delete a matrix
void matrix_delete(struct Matrix* m) {
    for (unsigned int i = 0; i < m->r; ++i)
        free(m->values[i]);
    free(m->values);
}
```

Utilisation (1/2)

```
int main(void) {
    srand(time(NULL));
    struct Matrix m1, m2;
    printf("Initializing m1:\n");
    matrix_initialize(&m1, 3, 5, true);
    matrix_print(&m1);
    printf("Initializing m2:\n");
    matrix_initialize(&m2, 3, 5, true);
    matrix_print(&m2);
    printf("Adding m2 to m1:\n");
    matrix_add(&m1, &m2); matrix_print(&m1);
    matrix_delete(&m1); matrix_delete(&m2);
    return 0;
}
```

Utilisation (2/2)

Résultat (les valeurs peuvent varier):

Initializing m1:

```
[ 7.07  9.70 -6.46 -4.45  1.04 ]  
[ -5.21 -1.31 -6.26 -8.26  0.70 ]  
[ 1.93 -8.22 -0.17 -8.96  8.88 ]
```

Initializing m2:

```
[ -0.74  1.29  1.24 -8.09 -3.17 ]  
[ -1.95 -6.93 -2.18  1.96 -4.75 ]  
[ 4.34  3.63  0.78 -5.20  7.19 ]
```

Adding m2 to m1:

```
[ 6.33 10.99 -5.22 -12.54 -2.14 ]  
[ -7.15 -8.24 -8.44 -6.30 -4.05 ]  
[ 6.26 -4.59  0.61 -14.16 16.07 ]
```

Arbres binaires de recherche

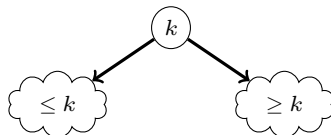
Arbres binaires de recherche

Arbre binaire

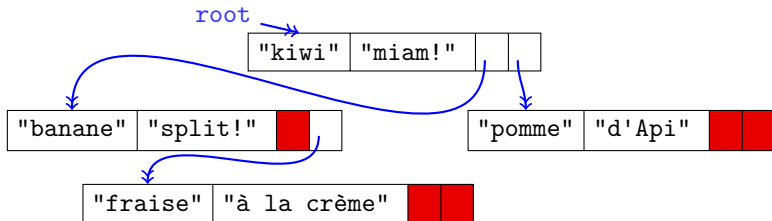
- Ensemble de noeuds organisés de façon **hiérarchique**
- Chaque noeud référence **deux** enfants
- Possiblement **vides**

Arbre binaire de recherche (ABR)

- Chaque noeud est identifié par une **clé**
- Choisie dans un ensemble de clés **totalemtent ordonné**
- Et contient une **valeur**
- **Invariant:**



Représentation



```
// A tree node
struct TreeNode {
    char* key;           // Key
    char* value;         // Value
    struct TreeNode* left; // Left subtree
    struct TreeNode* right; // Right subtree
};

// A tree map
typedef struct {
    struct TreeNode* root; // Root of tree
} Treemap;
```

Interface

```
// Initialize an empty tree map
void treemap_initialize(Treemap* t);

// Return the value associated with the given key in a tree map
char* treemap_get(const Treemap* t, const char* key);

// Set the value for the given key in a tree map
void treemap_set(Treemap* t, const char* key, const char* value);

// Indicate if a key exists in a tree map
bool treemap_has_key(const Treemap* t, const char* key);

// Print a tree map to stdout
void treemap_print(const Treemap* t);

// Delete a tree map
void treemap_delete(Treemap* t);
```

Récupérer un noeud

```
struct TreeNode* treemap_get_node(const struct TreeNode* node,
                                   const char* key) {
    if (node == NULL) {
        return NULL;
    } else {
        int cmp = strcmp(key, node->key);
        if (cmp == 0)
            return (struct TreeNode*)node;
        else if (cmp < 0)
            return treemap_get_node(node->left, key);
        else
            return treemap_get_node(node->right, key);
    }
}
```

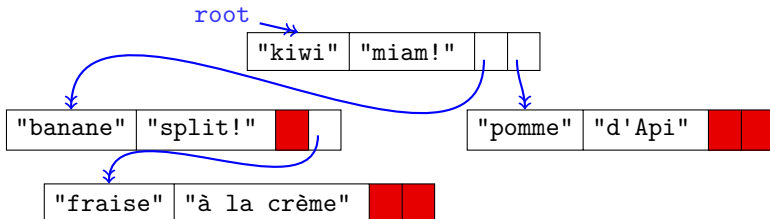
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```

Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```

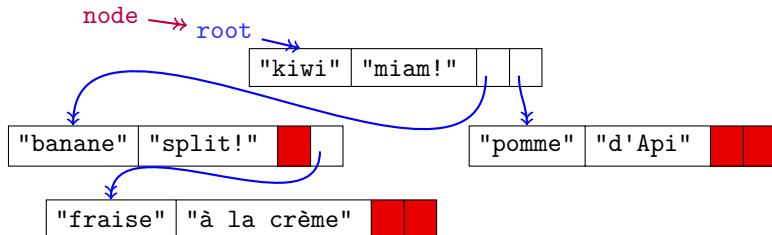
On souhaite insérer
la clé melon



Insérer un noeud

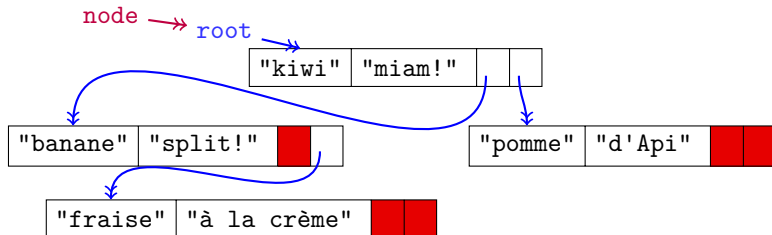
```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```

On souhaite insérer
la clé melon



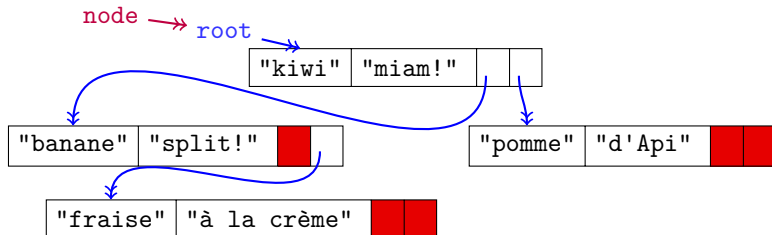
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



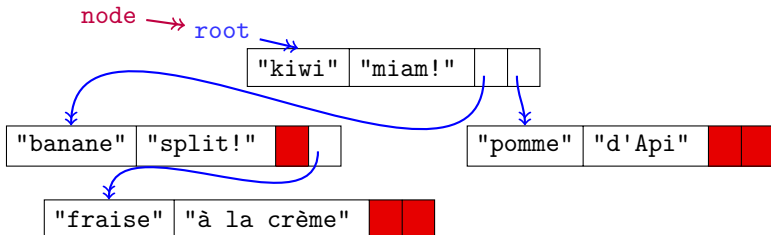
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



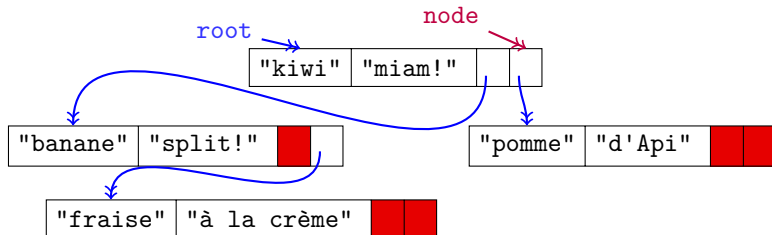
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



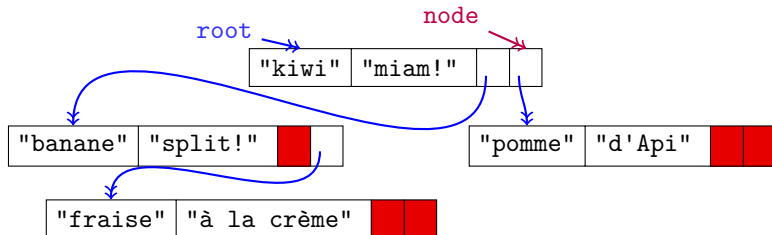
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



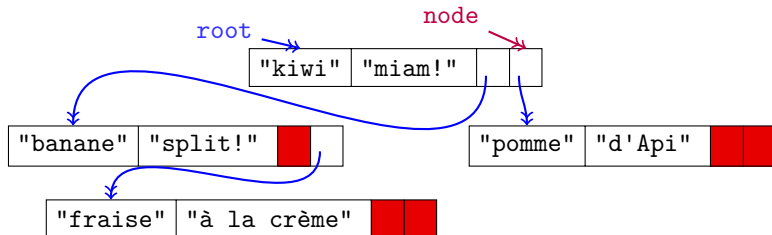
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,  
                        const char* key,  
                        const char* value) {  
    if (*node == NULL) {  
        *node = malloc(sizeof(struct TreeNode));  
        (*node)->key = strdup(key);  
        (*node)->value = strdup(value);  
        (*node)->left = NULL;  
        (*node)->right = NULL;  
    } else if (strcmp(key, (*node)->key) < 0) {  
        treemap_insert_node(&(*node)->left, key, value);  
    } else {  
        treemap_insert_node(&(*node)->right, key, value);  
    }  
}
```



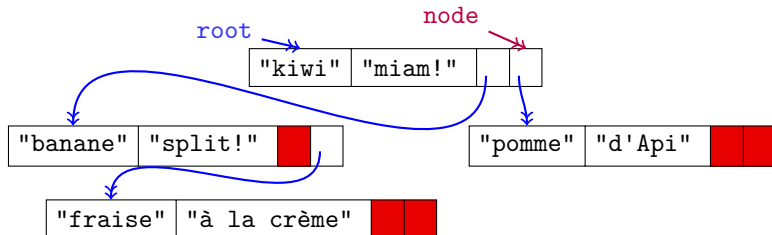
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



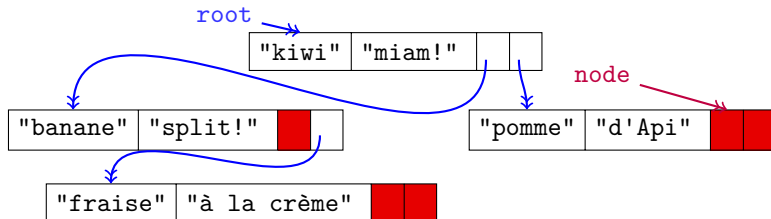
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



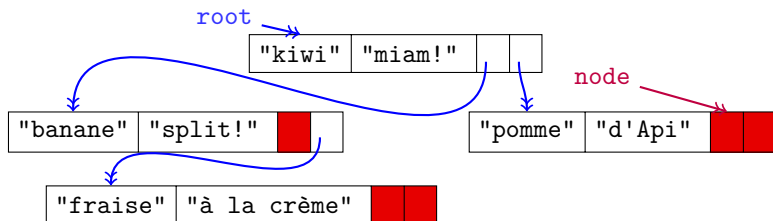
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



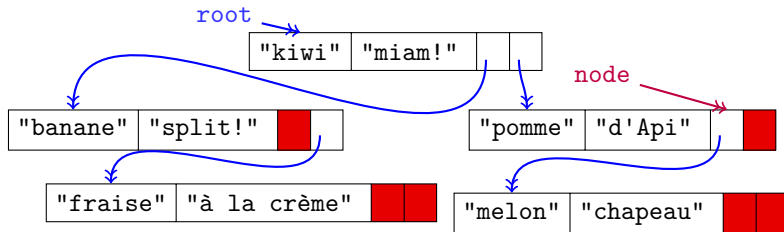
Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,  
                        const char* key,  
                        const char* value) {  
    if (*node == NULL) {  
        *node = malloc(sizeof(struct TreeNode));  
        (*node)->key = strdup(key);  
        (*node)->value = strdup(value);  
        (*node)->left = NULL;  
        (*node)->right = NULL;  
    } else if (strcmp(key, (*node)->key) < 0) {  
        treemap_insert_node(&(*node)->left, key, value);  
    } else {  
        treemap_insert_node(&(*node)->right, key, value);  
    }  
}
```



Insérer un noeud

```
void treemap_insert_node(struct TreeNode** node,
                        const char* key,
                        const char* value) {
    if (*node == NULL) {
        *node = malloc(sizeof(struct TreeNode));
        (*node)->key = strdup(key);
        (*node)->value = strdup(value);
        (*node)->left = NULL;
        (*node)->right = NULL;
    } else if (strcmp(key, (*node)->key) < 0) {
        treemap_insert_node(&(*node)->left, key, value);
    } else {
        treemap_insert_node(&(*node)->right, key, value);
    }
}
```



Implémentation (1/3)

```
void treemap_initialize(Treemap* t) {
    t->root = NULL;
}

char* treemap_get(const Treemap* t, const char* key) {
    struct TreeNode* node = treemap_get_node(t->root, key);
    return node == NULL ? NULL : node->value;
}

bool treemap_has_key(const Treemap* t, const char* key) {
    return treemap_get_node(t->root, key) != NULL;
}

void treemap_set(Treemap* t, const char* key, const char* value) {
    struct TreeNode* node = treemap_get_node(t->root, key);
    if (node != NULL) {
        free(node->value);
        node->value = strdup(value);
    } else {
        treemap_insert_node(&(t->root), key, value);
    }
}
```

Implémentation (2/3)

```
void treemap_print(const Treemap* t) {
    printf("TreeMap {\n");
    treemap_print_node(t->root);
    printf("}\n");
}

void treemap_print_node(const struct TreeNode* node) {
    if (node != NULL) {
        treemap_print_node(node->left);
        printf("  %s: %s\n", node->key, node->value);
        treemap_print_node(node->right);
    }
}
```

Implémentation (2/3)

```
void treemap_delete(Treemap* t) {
    treemap_delete_node(t->root);
}

void treemap_delete_node(struct TreeNode* node) {
    if (node != NULL) {
        treemap_delete_node(node->left);
        treemap_delete_node(node->right);
        free(node->key);
        free(node->value);
        free(node);
    }
}
```

Utilisation (1/2)

```
int main() {
    Treemap t;
    treemap_initialize(&t);
    treemap_set(&t, "firstname", "Doina");
    treemap_set(&t, "lastname", "Precup");
    treemap_set(&t, "city", "Montreal");
    treemap_set(&t, "province", "Quebec");
    treemap_set(&t, "country", "Canada");
    treemap_set(&t, "position", "DeepMind");
    printf("Printing the tree map\n");
    treemap_print(&t);
    printf("Get \"firstname\": %s\n", treemap_get(&t, "firstname"));
    printf("Get \"province\": %s\n", treemap_get(&t, "province"));
    printf("Get \"position\": %s\n", treemap_get(&t, "position"));
    printf("Changing country to Romania\n");
    treemap_set(&t, "country", "Romania");
    printf("Get \"country\": %s\n", treemap_get(&t, "country"));
    printf("Printing the tree map\n"); treemap_print(&t);
    treemap_delete(&t);
}
```

Utilisation (2/2)

Résultat:

Printing the tree map

```
TreeMap {  
  city: Montreal  
  country: Canada  
  firstname: Doina  
  lastname: Precup  
  position: DeepMind Montreal Head  
  province: Quebec  
}
```

Get "firstname": Doina

Get "province": Quebec

Get "position": DeepMind Montreal Head

Changing country to Romania

Get "country": Romania

Printing the tree map

```
TreeMap {  
  city: Montreal  
  country: Romania  
  firstname: Doina  
  lastname: Precup  
  position: DeepMind Montreal Head  
  province: Quebec  
}
```